Exercise 7

Sketch the following surfaces:

(a) $z = r^2$

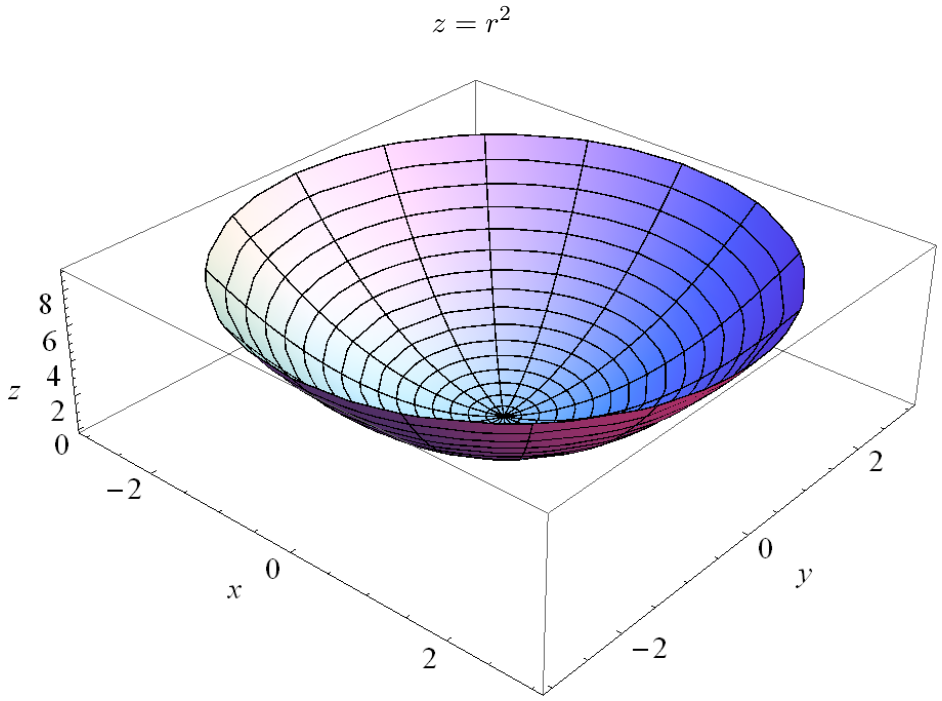
(b) $\rho = 4 \csc\phi \sec\theta$

(c) $r = 4 \sin\theta$

(d) $\rho \sin\phi = 2$

Solution

Part (a)



Part (b)

$$\rho = 4 \csc\phi \sec\theta$$

Write the equation in terms of sine and cosine.

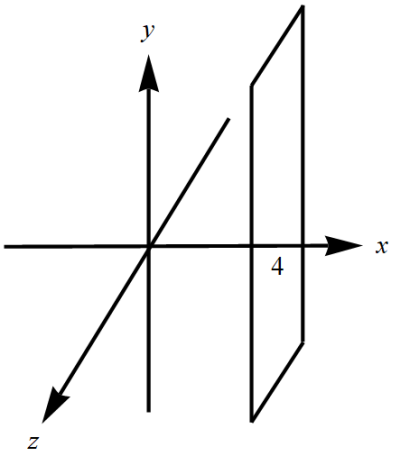
$$\rho = \frac{4}{\sin\phi \cos\theta}$$

Multiply both sides by $\sin\phi\cos\theta$.

$$\rho \sin\phi \cos\theta = 4$$

Substitute $x = \rho \sin\phi \cos\theta$.

$$x = 4$$



Part (c)

$$r = 4\sin\theta$$

Change this to Cartesian coordinates by substituting $r = \sqrt{x^2 + y^2}$ and $y = r\sin\theta$.

$$\sqrt{x^2 + y^2} = 4\left(\frac{y}{\sqrt{x^2 + y^2}}\right)$$

Multiply both sides by $\sqrt{x^2 + y^2}$.

$$x^2 + y^2 = 4y$$

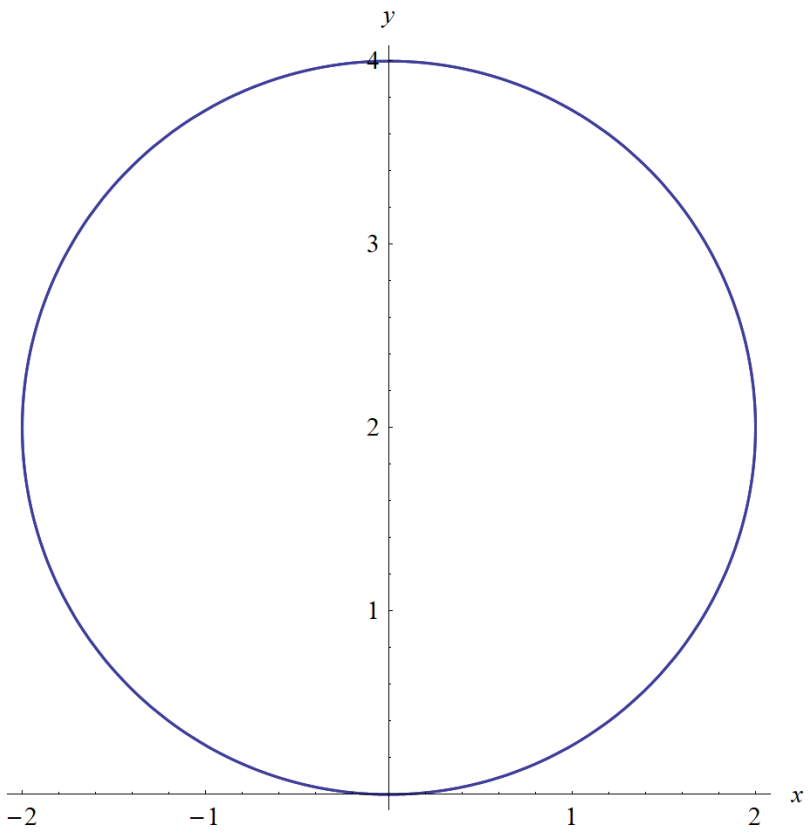
Bring $4y$ to the left side and complete the square.

$$x^2 + y^2 - 4y = 0$$

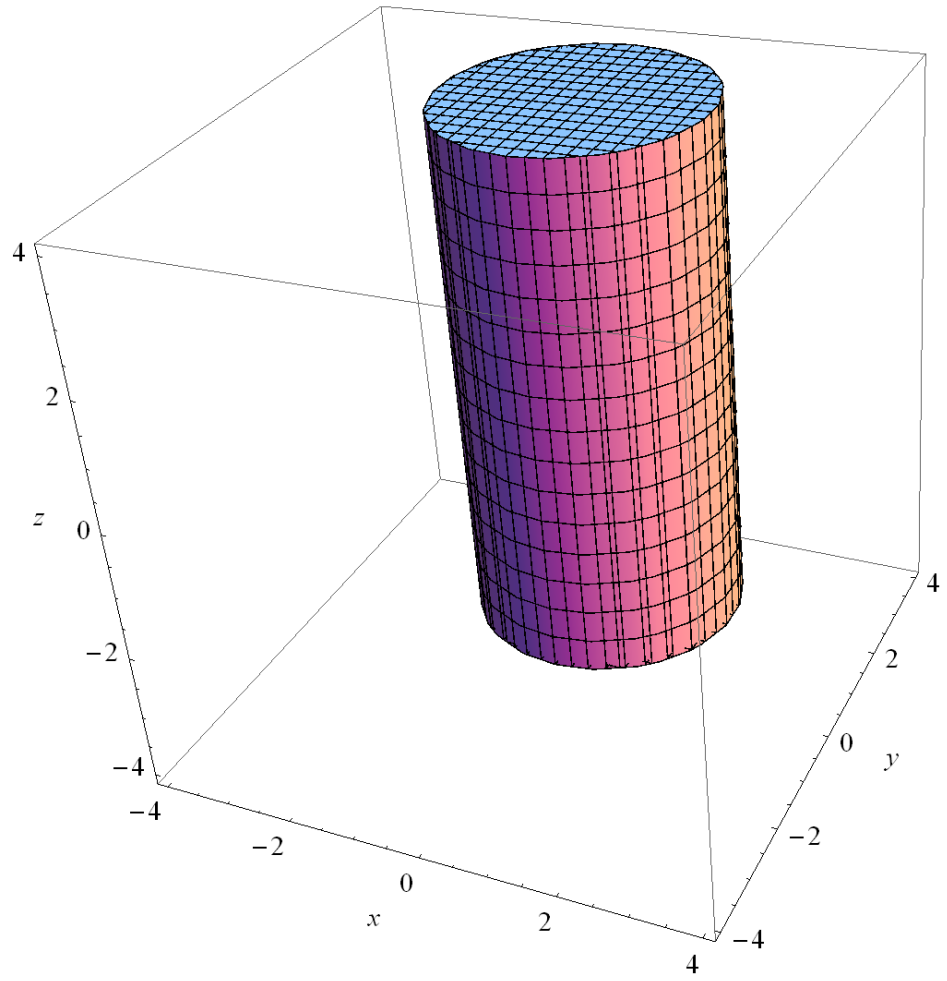
$$x^2 + y^2 - 4y + 4 = 4$$

$$x^2 + (y - 2)^2 = 4$$

This is a circle centered at $(0, 2)$ with a radius of 2.



In three dimensions this is actually a cylinder that extends indefinitely in the z-direction.



Part (d)

$$\rho \sin\phi = 2$$

Square both sides.

$$\rho^2 \sin\phi^2 = 4$$

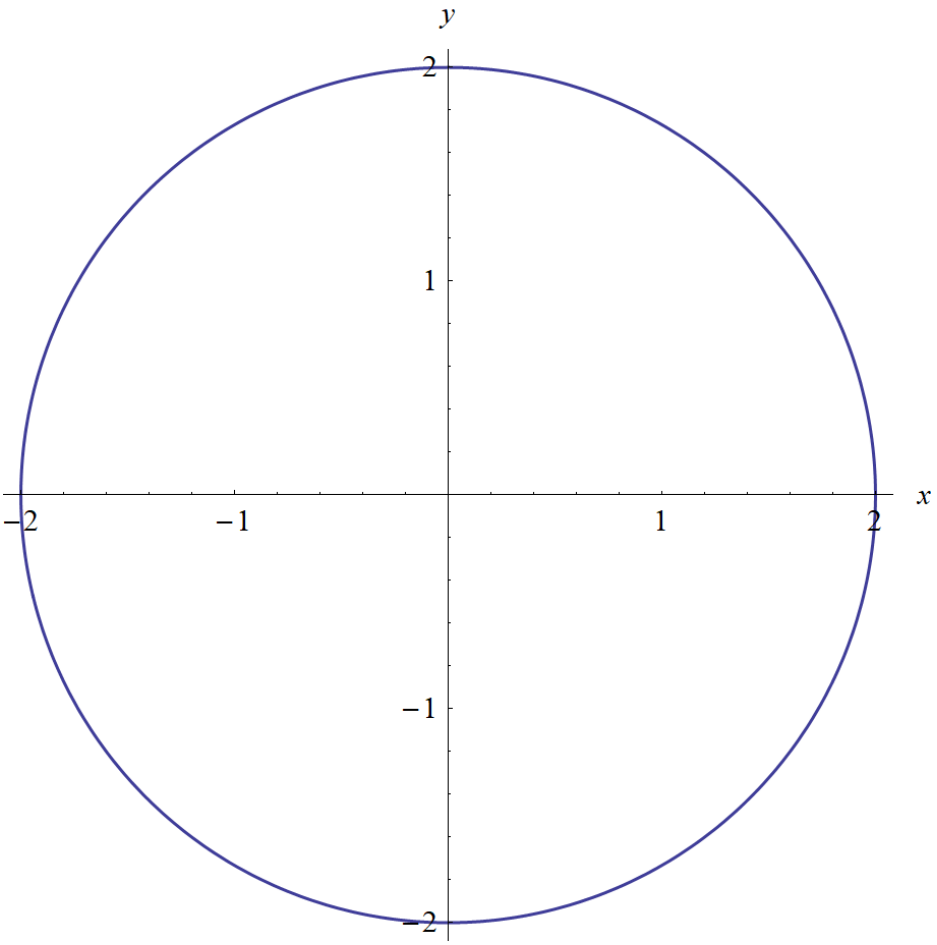
$$\rho^2 \sin\phi^2(\cos^2\theta + \sin^2\theta) = 4$$

$$\rho^2 \sin\phi^2 \cos^2\theta + \rho^2 \sin\phi^2 \sin^2\theta = 4$$

Substitute $x = \rho \sin\phi \cos\theta$ and $y = \rho \sin\phi \sin\theta$.

$$x^2 + y^2 = 4$$

This is a circle centered at $(0, 0)$ with radius 2.



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